9/20/17 "Some people ask "Why?", I ask "Why not?"." -Anonymous

HW: "Multiplying Probability" homework section

AIM: How do we multiply probabilities?

Warm Up:

The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

$P(\text{Male}) = \frac{230}{490} = \approx 47\%$

different

NOT Independent

$P(\text{Male} \mid \text{Reality}) = \frac{70}{180} \approx 39\%$

Probabilities involving **single-stage experiments** are easy enough because only one thing is happening to affect the probability, i.e. you flip a coin once, you pick one person at random, or you pull one card out of a deck. Probabilities, both empirical and theoretical, become increasingly more complicated with **multi-stage experiments**, where more than one thing happens, i.e. you flip a coin three times. How we handle these types of probabilities actually comes from the conditional probability formula.

***Exercise* #1:** Given that the probability of event B occurring given event A has occurred is $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$ answer the following.

(a) Rewrite this formula, solving for $P(\text{A and B})$.

$$P(A) \cdot P(B \mid A) = \frac{P(A \text{ and } B)}{\cancel{P(A)}} \cdot \cancel{P(A)}$$

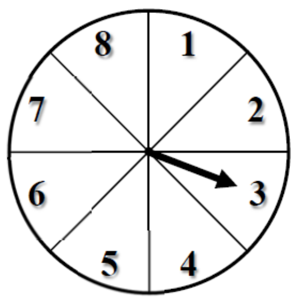
$$P(A) \cdot P(B \mid A) = P(A \text{ and } B)$$

(b) How could you write this formula if events A and B were **independent**?

$$P(A) \cdot P(B) = P(A \text{ and } B)$$

This rearrangement of the conditional probability formula gives us a useful tool for calculating the probability of events that occur in **multi-stage experiments**. You will easily be able to accomplish this if you systematically phrase the questions as unions of events (events connected by AND).

***Exercise* #2:** Consider the spinner shown below. The spinner is spun twice and the result is recorded.



(a) Are the outcomes of the two spins dependent or independent?

1st spin does not effect 2nd spin

(b) What is the probability that you will get an even on the first spin and a number greater than five on the second spin?

1st spin "Even" — 2nd spin "greater than 5"

$$\frac{4}{8} \times \frac{3}{8} = \boxed{\frac{12}{64}} \rightarrow \frac{3}{16}$$

(c) What is the probability that you will spin a prime number and a perfect square (in either order)? Note that this is more complex than (b).

* "And" "Both" $\longrightarrow$ Multiply

"OR" "Either" $\longrightarrow$ Add.

As experiments grow more complicated with more stages, theoretical probability becomes increasingly more complicated. It is especially important to note whether you are sampling **with or without replacement**.

20 total

***Exercise* #3:** A class consists of 12 girls and 8 boys. A group of three is picked to give a speech. If the students are picked at random, what is the probability that they all will be boys? Use the events below to show how you calculated your final answer.

P (1st is boy AND 2nd boy AND 3rd Boy)

Let: E_1 = Event that the first picked was a boy

E_2 = Event that the second picked was a boy

E_3 = Event that the third picked was a boy

$$\frac{8}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} = \frac{336}{6840} = \frac{14}{285} \approx \boxed{5\%}$$

The **multiplication property of probability** is crucial in many applications in engineering decision making.

***Exercise* #4:** Say that a power generating facility has three primary safety switches in case of an emergency. The probability that any one of these switches would fail is 5%. What is the probability all three will fail given that the switches are **independent** of one another?

1st and 2nd and 3rd

.0125%

$$(.05) \times (.05) \times (.05) = \boxed{.000125}$$

Many times when using the multiplication rule we need to be careful about how we frame the question. But, if we properly frame it in terms of AND and OR logical connectors, then the rules of probability will work out.

***Exercise* #5:** A company was determining the effectiveness of its warranty sales on computers. They took data on the number of customers who purchased warranties on two different brands of computers. If a customer was chosen at random, what is the probability they did not purchase a warranty?

	Percentage of Customers Purchasing	Percent of Those Who Purchased that Also Purchased Warranty
Type 1	68%	35%
Type 2	32%	56%